Fatigue design of CFRP strengthened steel members

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\begin{abstract}
Fatigue failure is brittle and sudden and is one of the main problems with steel members and connections. Fatigue failure is most likely to occur in steel members and connections, such as crane steel beams and welded connections of beams and columns, under cyclic tensile loading. When the fatigue loading of an old structure increases or when some short cracks develop in the structure, steps must be taken to prevent fatigue failure. Retrofitting existing structures instead of replacing them is the more economical and practical solution\cite{2}. Therefore, many studies have focused on the feasibility and efficiency of different strengthening methods, and many scholars have shown that carbon fiber reinforced polymer (CFRP) is effective and practical for reinforcing steel members under fatigue. Liu et al.\cite{3} explained the mechanism through which CFRP increases the fatigue life of cracked steel plates and then proposed a method based on fracture mechanics to predict the fatigue life. Nakamura et al.\cite{4} used CFRP to reinforce welded web gusset joints with fatigue cracks and found that the increase in the fatigue life was significant. Yang et al.\cite{5} quantified the behavior of the bonded steel–fiber reinforced polymer (FRP) interface and established a foundation for analyzing the behavior of steel-FRP hybrid plates under fatigue loading and static loading. Feng et al.\cite{6} studied the thermal effects on the fatigue behavior of cracked steel plates strengthened by CFRP sheets and found that at temperatures ranging from $-40 ^\circ C$ to $60 ^\circ C$, the fatigue lives of the strengthened samples were 2.0–3.4 times greater than those of the unstrengthened ones. Yu et al.\cite{7,8} used the boundary element method to analyze edge-cracked steel plates strengthened with CFRP laminates. Their results indicated that CFRP overlays can effectively slow crack growth and extend the fatigue life of edge-cracked steel plates regardless of the initial damage. Hu et al.\cite{9} analyzed the fatigue behavior of CFRP strengthened high-strength steel. Their study showed that CFRP strengthening can effectively increase the fatigue life (1.3–3.1 times) for all specimens except for those under very high stress range. Other studies have also shown that CFRP can be used to strengthen metal columns and girders\cite{10–14}. Recent studies have focused on new aspects related to this field. Zheng et al.\cite{15} studied debonding at the crack tip of a CFRP strengthened steel plate by digital image correlation technology. Wang et al.\cite{16} also applied this technology to study the bond behavior between CFRP plates and steel substrates under fatigue.
\end{abstract}

\section{Introduction}

Fatigue failure is one of the most serious failure types for steel structures and may result in casualties and considerable economic loss\cite{1}. Fatigue failure is most likely to occur in steel members and connections, such as crane steel beams and welded connections of beams and columns, under cyclic tensile loading. When the fatigue loading of an old structure increases or when some short cracks develop in the structure, steps must be taken to prevent fatigue failure. Retrofitting existing structures instead of replacing them is the more economical and practical solution\cite{2}. Therefore, many studies have focused on the feasibility and efficiency of different strengthening methods, and many scholars have shown that carbon fiber reinforced polymer (CFRP) is effective and practical for reinforcing steel members under fatigue. Liu et al.\cite{3} explained the mechanism through which CFRP increases the fatigue life of cracked steel plates and then proposed a method based on fracture mechanics to predict the fatigue life. Nakamura et al.\cite{4} used CFRP to reinforce welded web gusset joints with fatigue cracks and found that the increase in the fatigue life was significant. Yang et al.\cite{5} quantified the behavior of the bonded steel–fiber reinforced polymer (FRP) interface and established a foundation for analyzing the behavior of steel-FRP hybrid plates under fatigue loading and static loading. Feng et al.\cite{6} studied the thermal effects on the fatigue behavior of cracked steel plates strengthened by CFRP sheets and found that at temperatures ranging from $-40 ^\circ C$ to $60 ^\circ C$, the fatigue lives of the strengthened samples were 2.0–3.4 times greater than those of the unstrengthened ones. Yu et al.\cite{7,8} used the boundary element method to analyze edge-cracked steel plates strengthened with CFRP laminates. Their results indicated that CFRP overlays can effectively slow crack growth and extend the fatigue life of edge-cracked steel plates regardless of the initial damage. Hu et al.\cite{9} analyzed the fatigue behavior of CFRP strengthened high-strength steel. Their study showed that CFRP strengthening can effectively increase the fatigue life (1.3–3.1 times) for all specimens except for those under very high stress range. Other studies have also shown that CFRP can be used to strengthen metal columns and girders\cite{10–14}. Recent studies have focused on new aspects related to this field. Zheng et al.\cite{15} studied debonding at the crack tip of a CFRP strengthened steel plate by digital image correlation technology. Wang et al.\cite{16} also applied this technology to study the bond behavior between CFRP plates and steel substrates under fatigue.
loading, which is a critical and fundamental issue. Zheng et al. [17] applied a thermally activated shape memory alloy to achieve the pre-stressing of the CFRP, whose use is more efficient than that of CFRP without pre-stress. Aljabar et al. [18] analyzed a case of mixed tension and shear loading and found a shifting phenomenon in terms of crack propagation. Borrie et al. [19] conducted a large number of fatigue tests on CFRP patched pre-cracked steel plates after extreme environmental exposure and proved that CFRP reinforcement can be adapted even under extreme exposure scenarios. Many other studies focused on the repair of steel girders and bridges. For example, Yue et al. [20] studied the fatigue performance of CFRP reinforced steel crane girders and concluded that the CFRP layers can perfectly restrain the development of fatigue damage.
of fatigue fracture. Ghafoori et al. [21,22] proposed fatigue design criteria for strengthening metallic beams with bonded CFRP plates and suggested to apply pre-stress to the CFRP. Ali et al. [23] examined the crack growth associated with CFRP repairs to cracked bridge steels and firstly gave the da/dN versus SIF curves of bridge steel.

However, the existing fatigue design guides address only pure steel members, as summarized in [24], where fatigue design using various codes (AS 4100 (Standard Australia 1998), Eurocode 3 Part 1.9 (EC 3 2003) and BS 7608 (BSI 1993)) was presented. In addition, Zhao et al. [25] gave design recommendations for structural hollow sections under fatigue loading. Later, the International Institute of Welding (IIW) provided recommendations for the fatigue design of welded joints and components [26].

Based on the large number of studies on the fatigue behavior of CFRP strengthened steel members and the extensive design guidance for pure steel under fatigue loading, fatigue design of CFRP strengthened steel members is clearly needed. This paper proposes a fatigue design method and programs for CFRP strengthened steel members both with and without initial fatigue cracks. First, based on previous recommendations [26], a calculation method and strengthening method are proposed for steel members. A corresponding calculation method is then proposed for obtaining the reduced stress range of the strengthened steel (S_{with CFRP}), which is called the constant amplitude fatigue limit; and when the nominal stress range is below S_{with CFRP} = S_{without CFRP} 	imes \frac{1}{1 + \left( \frac{S}{S'} \right)^{\frac{1}{2}}}

\text{Step 1} Determine the partial safety factor γ_M.

\text{Step 2} Determine the fatigue loading, the design stress range and the design spectrum for the detail.

\text{Step 3} Determine the appropriate design category.

\text{Step 4} Determine the fatigue life using Eq. (1)

N_i = \left( \frac{S_{N_i}}{S'} \right) \times 2 \times 10^6 \times \frac{1}{S_i}

where \( S_i \) is the ith nominal stress range and \( N_i \) is the number of fatigue cycles predicted by Fatigue life prediction of Eq. (1) is based on the fatigue stress vs. number of cycles (S-N) curves, as schematically shown in Fig. 1 [24]. In this figure, \( S_{N_i} \) is the nominal stress range at 2 \times 10^6 cycles, which is called the detail category; \( S_{CA} \) is the nominal stress range at 5 \times 10^6 cycles, which is called the constant amplitude fatigue limit; and \( S_{CO} \) is the normal stress range at 10^8 cycles, which is called the cut-off limit. Constant amplitude loading assumes that no fatigue damage occurs when the nominal stress range is below \( S_{CA} \). For variable amplitude loading, the nominal stress range below \( S_{CO} \) does not contribute to the fatigue damage. The index (3 or 5) in Eq. (1) refers to the slope of the S-N curve.

\text{Step 5} Determine whether the fatigue limits are satisfied using fatigue damage accumulation.

\text{Step 6} If not, modify the structure or application or add reinforcement, and repeat Steps 1–5.

This method is used to calculate the fatigue life of steel members without initial fatigue cracks.

2.2. Reduced stress range due to CFRP strengthening

Without initial fatigue cracks, CFRP strengthened steel members will not experience debonding if proper surface treatment is performed [27]. The steel surface should be sandblasted and then cleaned with acetone [9]; vacuum assisted resin transfer molding (VARTM) can be used for overhead reinforcement. When applying CFRP on a steel surface that is not flat, VARTM [28] and a U-shaped hoop [20] are helpful.

The mechanism of CFRP strengthening steel plates or members in tension is shown in Fig. 2, and the nominal stress range of the strengthened steel (S_{with CFRP}) can be estimated using Eq. (2-a) or Eq. (2-b) or Eq. (3-a) or Eq. (3-b),

\begin{align*}
S_{\text{with CFRP}} &= S_{\text{without CFRP}} \times \frac{1}{1 + \left( \frac{S}{S'} \right)^{\frac{1}{2}}} \quad (2-a) \\
S_{\text{with CFRP}} &= S_{\text{without CFRP}} \times \frac{1}{1 + 2 \left( \frac{S}{S'} \right)^{\frac{1}{2}}} \quad (2-b) \\
S_{\text{with CFRP}} &= S_{\text{without CFRP}} \times \frac{1}{1 + \left( \frac{S}{S'} \right)^{\frac{1}{5}}} \quad (3-a) \\
S_{\text{with CFRP}} &= S_{\text{without CFRP}} \times \frac{1}{1 + 2 \left( \frac{S}{S'} \right)^{\frac{1}{5}}} \quad (3-b)
\end{align*}

where \( S_{\text{without CFRP}} \) is the nominal stress range of the unstrengthened steel, the ratio following S_{\text{without CFRP}} can be called the reduction factor, \( E \) is the elastic modulus, \( b \) is the width and \( t \) is the thickness. The subscripts c, s, f, and a represent an FRP composite made of FRP fiber and matched adhesive, steel, FRP fiber, and adhesive, respectively. Eq. (2-a) and Eq. (3-a) are for single-side reinforcement, and Eq. (2-b) and Eq. (3-b) are for double-side reinforcement.

For CFRP sheets, if we know the thickness of the adhesive, which we do when we calculate it after the reinforcement, Eq. (2-a) and Eq. (2-b)
should be used. In this case, $E_c$ can be obtained from Eq. (4),

$$E_c = \frac{E_{h} t_h + E_{l} t_l}{n(t_l + t_h)} \sum_{i=1}^{n} k_i$$  \hspace{1cm} (4)

where $n$ is the number of CFRP layers and $k_i$ is the factor between the strain in the $i^{th}$ layer of fiber and the strain of the steel [3]. The value of $k_i$ is based on previous experiments: for high modulus CFRP, $k_1 = 1.0$, $k_2 = 0.78$, and $k_3 = k_4 = k_5 = 0.56$; for normal modulus CFRP, $k_1 = 1.0$, $k_2 = 0.73$, and $k_5 = k_4 = k_3 = 0.17$ [29,30]. Before reinforcement, if we do not know the amount of adhesive, a conservative estimate is to assume that only the fiber is working; then, Eq. (3-a) and Eq. (3-b) can be used. In this case, if we apply more than one layer, the elastic modulus of the CFRP should also be reduced, as shown in Example 2.

For CFRP laminate, Eq. (3-a) and Eq. (3-b) are suggested.

### 2.3. Classification table after CFRP strengthening

There are 2 approaches for handling improved fatigue life design. One is to use the original Fatigue Category for the detail and to use the reduced stress range for design, and the second approach is to use the stress range calculated based on steel alone and to adopt a new Fatigue Category. The first approach is used in this paper since designers are familiar with the classification method for pure steel structures. Table 1 lists $b_h$, which also represents $b_l$ and $b_b$. The longitudinal fiber direction is indicated by “$a$”, and the location and direction of the stress range are indicated by “$b”. However, for some cases, such as Structure detail 6, the reduction factor is difficult to calculate. For the different structure details in Table 1, two grades are proposed. Grade 1 is for those details that have been explicitly tested in fatigue tests [31–36], for which the design method could be applied with experimental support, and Grade 2 is for those details that have not been explicitly tested in fatigue tests, for which the Fatigue Category is purely based on the reduced stress range described in Section 2.2.

### 2.4. Accumulated damage

According to the linear damage calculation using the “Palmgren-Miner” Rule, for fatigue verification, it has to be shown that:

$$D = \sum_{i=1}^{n} \frac{n_i}{N_i} \leq 1.0$$  \hspace{1cm} (5)

where $D$ is the linear damage sum, named the fatigue damage accumulation, and $n_i$ is the number of fatigue cycles under the design load stress range in load spectrum block $i$. If the maximum design stress range in the load spectrum is less than $S_{CD}$, no further damage calculation is necessary. Two examples are shown in Section 4.1.

### 3. Reinforcement for steel members with initial fatigue cracks

#### 3.1. Fracture mechanics

In fracture mechanics [37,38], the stress intensity factor (SIF) indicates the intensity of the stress field at the crack tip. For plates with a crack of $2a$ (as shown in Fig. 3), the SIF is equal to:

$$K = \frac{Fa}{\sqrt{a}}$$  \hspace{1cm} (6)

where $a$ is the half crack length, $f$ is the nominal stress of the steel plate, and $f$ is the correction factor for real conditions (e.g., finite geometry, temperature).

Based on the SIF, Paris and Erdogan proposed the Paris Law [39], which gives the relationship of the crack propagation rate $da/dN$ and the range of SIF $\Delta K (= K_{\text{max}} - K_{\text{min}})$.

$$\frac{da}{dN} = C(\Delta K)^m$$  \hspace{1cm} (7)

where $C$ and $m$ are material parameters and $N$ is the number of fatigue cycles. Considering the crack opening stress [40] and the threshold value of the range of the SIF, the linear elastic fracture method (LEFM) gives the basic calculation method for obtaining the fatigue life, as shown in Eq. (8),

$$N_p = \int_{a_i}^{a_f} \frac{1}{C(\Delta K_{\text{eff}})^m - (\Delta K_{th})^m} \, da$$  \hspace{1cm} (8)

where $a_i$ is the initial half crack length, $a_f$ is the final half crack length, $N_p$ is the predicted fatigue life, $\Delta K_{\text{eff}}$ is the effective range of the SIF, and $\Delta K_{th}$ is the threshold value of the range of the SIF. If the steel members do not work under ambient temperature, we need to consider the thermal effects on the fatigue life prediction by introducing modified coefficients [6].

#### 3.2. Reduced stress range due to CFRP strengthening

CFRP can reduce the crack growth and extend the fatigue life in three different ways: (i) by reducing the effective stress range around the crack tip; (ii) by reducing the crack opening displacement (COD); and (iii) by promoting crack closure [41]. According to our experimental data, the reduction of the stress range has a major effect on the crack propagation and fatigue life [6,9]. Additionally, the last two ways are difficult to calculate under different situations [42]. Thus, we only consider the first way in design, as it is safe and simple. However, because crack closure is emphasized when the CFRP strips are prestressed, the effect of crack closure should be considered when we design prestressed-CFRP reinforced steel structures.

The following analysis is for the CFRP strengthening of steel plates or members in tension. Due to its high tensile strength, CFRP is used to share tensile stress with the original tensile elements. Because CFRP is strain-compatible with the steel through adhesive, it can bear some of the nominal stress, reducing the stress in the steel. As a result, the stress concentration near the crack decreases, as shown in Fig. 4; thus, CFRP
helps to slow the crack growth rate and thus extend the fatigue life. The nominal stress range shared with the steel ($\Delta \sigma_{\text{with CFRP}}$) can be calculated using Eq. (9), where $\Delta \sigma_{\text{without CFRP}}$ is the nominal stress range of the unstrengthened steel and $\Delta K_{\text{eff,s}}$ (the effective range of SIF of strengthened steel) is obtained from $\Delta \sigma_{\text{with CFRP}}$. In addition, the predicted fatigue life of the strengthened steel $N_{p,s}$ can be obtained from Eq. (10):

$$
\Delta \sigma_{\text{with CFRP}} = \Delta \sigma_{\text{without CFRP}} \times \frac{E_t \left( b_1 - 2a \right)}{E_t \left( b_1 - 2a \right) + 2E_b t_c}.
$$

$$
\Delta \sigma_{\text{with CFRP}} = \Delta \sigma_{\text{without CFRP}} \times \frac{E_t \left( b_1 - 2a \right)}{E_t \left( b_1 - 2a \right) + 2E_b t_c}.
$$

$$
N_{p,s} = \int_{a_0}^{a_1} \frac{1}{C((\Delta K_{\text{eff,s}}) - (\Delta K_{\text{th,s}}))} \, da
$$

Table 1
Examples of structure details and $S_{DC}$.

<table>
<thead>
<tr>
<th>Structure detail</th>
<th>Reinforcement layout</th>
<th>Description</th>
<th>$S_{DC}$</th>
<th>Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Unwelded parts of a component</td>
<td>(by CFRP sheet or laminate)</td>
<td>Rolled or extruded products, components with machined edges, seamless hollow sections</td>
<td>160</td>
<td>Grade 2</td>
</tr>
<tr>
<td></td>
<td>(by CFRP sheet or laminate)</td>
<td>Machine gas cut or sheared material with subsequent dressing, no cracks by inspection, no visible imperfections</td>
<td>140</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(by CFRP sheet or laminate)</td>
<td>Machine thermally cut edges, corners removed, no cracks by inspection</td>
<td>125</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(by CFRP sheet)</td>
<td>Manually thermally cut edges, free from cracks and severe notches</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(by CFRP sheet)</td>
<td>Manually thermally cut edges, uncontrolled, no notch deeper than 0.5 mm</td>
<td>80</td>
<td></td>
</tr>
<tr>
<td>2. Butt welds, transverse loaded</td>
<td>(by CFRP sheet or laminate)</td>
<td>Transverse loaded butt weld (X-groove or V-groove) ground flush to plate, 100% non-destructive testing (NDT)</td>
<td>112</td>
<td>Grade 2</td>
</tr>
<tr>
<td></td>
<td>(by CFRP sheet)</td>
<td>Transverse butt weld made in shop in flat position, NDT weld reinforcement &lt; 0.1 thickness</td>
<td>90</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(by CFRP sheet)</td>
<td>Transverse butt weld not satisfying conditions of 212, NDT</td>
<td>80</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(by CFRP sheet)</td>
<td>Transverse butt weld, welded on non-fusible temporary backing, root crack</td>
<td>80</td>
<td>Grade 2</td>
</tr>
<tr>
<td></td>
<td>(by CFRP sheet)</td>
<td>Transverse butt weld on permanent backing bar</td>
<td>71</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(by CFRP sheet)</td>
<td>Transverse butt welds welded from one side without backing bar, full penetration: Root checked by appropriate NDT</td>
<td>71</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(by CFRP sheet)</td>
<td>No NDT</td>
<td>36</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(by CFRP sheet)</td>
<td>Transverse butt weld ground flush, NDT, with transition in thickness and width:</td>
<td>112</td>
<td>Grade 2</td>
</tr>
<tr>
<td></td>
<td>(by CFRP sheet)</td>
<td>slope 1:5</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(by CFRP sheet)</td>
<td>slope 1:3</td>
<td>90</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(by CFRP sheet)</td>
<td>slope 1:2</td>
<td>90</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(by CFRP sheet)</td>
<td>Transverse butt weld made in shop, welded in flat position, weld profile controlled, NDT, with transition in thickness and width:</td>
<td>90</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(by CFRP sheet)</td>
<td>slope 1:5</td>
<td>80</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(by CFRP sheet)</td>
<td>slope 1:3</td>
<td>72</td>
<td></td>
</tr>
<tr>
<td>Grade</td>
<td>Description</td>
<td>Value</td>
<td></td>
<td></td>
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<tr>
<td>-------</td>
<td>-----------------------------------------------------------------------------</td>
<td>-------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>71</td>
<td>Transverse butt weld splice in circular hollow section, welded from one side, full penetration, potential failure from root:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>root inspected by NDT</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>no NDT</td>
<td>36</td>
<td></td>
<td></td>
</tr>
<tr>
<td>125</td>
<td>Automatic longitudinal seam welds:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>without stop/start positions in hollow sections</td>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td>with stop/start positions</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>112</td>
<td>Longitudinal butt weld, both sides ground flush parallel to load direction, proven free from significant defects by appropriate NDT</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>90</td>
<td>Longitudinal butt weld:</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>without stop/start positions, NDT</td>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td>with stop/start positions</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>125</td>
<td>Continuous automatic longitudinal fully penetrated K-butt weld without stop/start positions (based on stress range in flange) NDT</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>112</td>
<td>Continuous automatic longitudinal double-sided fillet weld without stop/start positions (based on stress range in flange)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>90</td>
<td>Continuous manual longitudinal fillet or butt weld (based on stress range in flange)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>71</td>
<td>Longitudinal butt weld, fillet weld or intermittent weld with cope holes (based on normal stress in flange $\sigma$ and shear stress in web $\tau$ at weld ends), cope holes not higher than 40% of web:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>63</td>
<td>$\tau/\sigma = 0$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>56</td>
<td>$\tau/\sigma = 0.0 - 0.2$</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>50</td>
<td>$\tau/\sigma = 0.2 - 0.3$</td>
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<tr>
<td>45</td>
<td>$\tau/\sigma = 0.3 - 0.4$</td>
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<tr>
<td>40</td>
<td>$\tau/\sigma = 0.4 - 0.5$</td>
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</tr>
<tr>
<td>36</td>
<td>$\tau/\sigma &gt; 0.6$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>80</td>
<td>Cruciform joint or T-joint, K-butt welds, full penetration, weld toes ground, potential failure from weld toe</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>90</td>
<td>Single-sided T-joints and cruciform joints without misalignment</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>71</td>
<td>Cruciform joint or T-joint, K-butt welds, full penetration, potential failure from weld toe</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>80</td>
<td>Single-sided T-joints and cruciform joints without misalignment</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
5. Non-load-carrying attachments

- Transverse non-load-carrying attachment, not thicker than main plate:
  - K-butt weld, toe ground: 100
  - Two-sided fillets, toe ground: 100
  - Fillet weld(s), as welded: 80
  - Thicker than main plate: 71
  - Grade: 2

- Transverse stiffener welded on girder web or flange, not thicker than main plate:
  - K-butt weld, toe ground: 100
  - Two-sided fillets, toe ground: 100
  - Fillet weld(s), as welded: 80
  - Thicker than main plate: 71
  - Grade: 1
  - References: [31, 32, 33]

- Non-load-carrying rectangular or circular flat studs, pads or plates:
  - $L \leq 50$ mm: 80
  - $L > 50$ and $\leq 150$ mm: 71
  - $L > 150$ and $\leq 300$ mm: 63
  - $L > 300$ mm: 50
  - Grade: 2

- In-plane or out-of-plane longitudinal gusset welded to plate or beam flange edge, gusset length $l$:
  - $1 < 150$ mm: 50
  - $1 < 300$ mm: 45
  - $1 > 300$ mm: 40
  - Grade: 1
  - References: [35]

- Longitudinal flat side gusset welded on edge of plate or beam flange, radius transition ground:
  - $r > 150$ or $r/w > 1/3$: 90
  - $1/6 < r/w < 1/3$: 71
  - $r/w < 1/6$: 50
  - Grade: 2

6. Lap joints

- Transverse loaded lap joint with fillet welds:
  - Fatigue of parent metal: 63
  - Fatigue of weld throat: 45
  - Grade: 2

(by CFRP sheet)
3.3. Finite element analysis to calculate the interaction of debonding and crack propagation

3.3.1. Effect of SIF on debonding

The SIF of the steel in a CFRP strengthened steel component is a comprehensive parameter that describes the intensity of the local stress field at the crack tip considering the following parameters: the crack configuration, the loading conditions, and the geometric and material parameters of the steel plate, adhesive, and CFRP. As a result, the crack propagation can be reflected by the SIF.

According to Diab et al. [43], for a concrete-CFRP interface, the debonding evolution under fatigue can be divided into three stages. It is assumed that this rule is also true for a steel-CFRP interface, as shown in Fig. 5.

(1) In State (i), when $K_{\text{max}}$ is sufficiently small, no debonding occurs. Point A indicates when micro-debonding begins; this point corresponds to $K_{\text{debond}}$, which means that when $K_{\text{max}}$ reaches $K_{\text{debond}}$, micro-debonding starts. At this time, the half crack length is $a_d$.

(2) State (ii) is the transition period. Point B indicates when the half debonding length $b_d$ reaches the half crack length $a$. Before Point B, because $a > b_d$, the propagation of the debonding is fast; after Point B, it becomes almost equal to the crack propagation. Point C represents when the micro-debonding becomes macro-debonding and stable propagation occurs.

(3) In State (iii), when the SIF is relatively large, debonding propagates stably over a long period of time and finally develops rapidly due to an insufficient bonding length. The debonding propagation is affected by the SIF, as discussed below. Based on previous experiments, in State (iii), the debonding area was visually observed after the fatigue test [35] and was measured using optical speckle interferometry and C-scans during the test [27, 44]. The results
showed that elliptical debonding occurred around the crack tip. In addition, due to the opening and ovalization of the crack tip, as shown in Fig. 6 [45], an elliptical debonding area is reasonable. The stress variation of the adhesive in finite element modeling also indicates that elliptical debonding will occur around the crack tip, as shown in Fig. 7. Therefore, the equation by Colombi et al. [44] was adopted in this paper for determining the debonding area, as shown in Eq. (11). The debonding area is assumed to be an ellipse with a semimajor axis of $b_0$ and a semiminor axis of $c_0$; $R_p$ is the size of the plastic zone. Based on the Irwin model, it has been proven that the value of $R_p$ is small [46], so Eq. (11) can be simplified to Eq. (12). Therefore, the stable development of the debonding area is a changing ellipse controlled by Eq. (12).

$$b_0 = a + R_p, \quad c_0 = b_0/5, \quad R_p = 1/2\pi \times (K/f_s)^2$$  \hspace{1cm} (11)

$$b_3 = a, \quad c_3 = b_3/5$$  \hspace{1cm} (12)

It is necessary to obtain the value of $K_{\text{debond}}$. When the strain of the adhesive reaches its maximum value $\varepsilon_{a,\text{max}}$, as shown in Fig. 7, the energy absorbed by the adhesive reaches its ultimate value, so micro-debonding occurs, and $K_{\text{max}}$ reaches $K_{\text{debond}}$. This means that debonding at the cracked section is achieved when the shear stress or strain exceeds the allowable limit [41]. Three-dimensional FE models were assembled and analyzed using the commonly used ANSYS (ANSYS Inc., Canonsburg, PA) FE software [47]. In the FE models (shown in Fig. 7(a)), the CFRP plate was assumed to be orthotropic, and the steel and adhesive were assumed to be isotropic. The relevant properties of the CFRP, steel, and adhesive are listed in Table 2. The CFRP composite was meshed with SHELL 63 elements. The steel plate was simulated by SOLID45 elements, while the adhesive layer was modeled by non-linear spring elements COMBINE 39. The SIF is obtained through the virtual crack closure method (VCCM) [48]. The FE analysis showed a relationship between $K_{\text{max}}$ and the strain of the adhesive (simulated by a spring element) at the crack tip $\varepsilon_a$ as given in Eq. (13); thus, $K_{\text{debond}}$ can be defined as shown in Eq. (14). When $K_{\text{max}}$ reaches $K_{\text{debond}}$, micro-debonding at the crack tip occurs. Eq. (13) and Eq. (14) are applicable for normal strength steel plates of 8-mm to 12-mm thickness, CFRP (sheet or laminate) with a modulus ranging from 100 GPa to 500 GPa, a commonly used adhesive such as E2500S from KONY BOND and the Araldite series, and single-sided or double-sided CFRP reinforcement.

$$\varepsilon_a = 7.37 \times 10^{-5} \times K_{\text{max}} - 1.38 \times 10^{-3}$$  \hspace{1cm} (13)

$$K_{\text{debond}} = 1.357 \times 10^4 \times (\varepsilon_{a,\text{max}} + 1.38 \times 10^{-3})$$  \hspace{1cm} (14)

### 3.3.2. Effect of debonding on the SIF

In the following analysis, for calculation efficiency, the adhesive is stimulated as the elastic spring element COMBINE 14, and the debonding zone is simulated by deleting spring elements according to the configuration given by Eq. (12). Only static loading was applied to study the effect of debonding on the SIF. A coefficient $d$ is defined as the ratio of the SIF under any nominal stress $\sigma$ considering the debonding at the crack tip ($K_{\text{with,}\sigma}$) to that without considering debonding at the crack tip ($K_{\text{no,}\sigma}$), as shown in Eq. (15):

$$d = K_{\text{with,}\sigma}/K_{\text{no,}\sigma}$$  \hspace{1cm} (15)

It maintains the same value under fatigue loading. In general, $d$ may be influenced by the axial stiffness of the FRP composite consisting of FRP fiber and matched adhesive ($S_t$), the axial stiffness of the adhesive between the steel and the FRP composite ($S_a$), and the axial stiffness of the steel plate ($S_s$); it may also be affected by the nominal stress $\sigma$ and the damage ratio $\beta$. The possible influential factors are defined in Eqs. (16)–(18). The effects of $\beta$, the stiffness ratios (i.e., $S_t/S_s$ and $S_a/S_s$) and $\sigma$ are shown in Fig. 8(a)–(d). $\beta$ has a relatively significant influence on $d$, but the influences of the stiffness ratios and the nominal stress can be neglected within the validity ranges. Thus, the quadratic regression method gives Eq. (18) for calculating $d$.

$$\beta = 2a/b_1 \quad (0 < \beta < 1)$$  \hspace{1cm} (16)

$$S_t = E_t h_t t_s, \quad S_a = E_a h_a t_a, \quad S_s = E_s h_s t_s \quad (0.07 < S_t/S_s < 0.13, \quad 10^{-3} < S_a/S_s < 10^{-6})$$  \hspace{1cm} (17)

$$d = 0.96\beta^2 - 0.4\beta + 1.1$$  \hspace{1cm} (18)

### 3.4. Enlarged SIF due to debonding

As Eq. (10) fails to consider the effect of debonding on the fatigue life, a modified Paris Law considering debonding was proposed, as shown in Eqs. (19)–(21), where $N_p,\varepsilon,\sigma$ is the predicted fatigue life of the...
strengthened steel considering debonding. These equations divide the whole process of crack propagation into two segments: from \( a_i \) to \( a_d \) and from \( a_d \) to \( a_f \). For the latter phase, \( d \) is added to enlarge the range of SIF because micro-debonding has occurred. The validity ranges of the parameters are shown in Eqs. (16) and (17), and the applicable conditions are the same as those for Eqs. (13) and (14).

\[
K_{\text{debond}} = \sigma \times \frac{E_A (b_h - 2a_d)}{E_J (b_h - 2a_d) + E_J h c} \times f \times \sqrt{\beta a_d}
\]

(19-a)

\[
K_{\text{debond}} = \sigma \times \frac{E_J (b_h - 2a_d)}{E_J (b_h - 2a_d) + 2E_J h c} \times f \times \sqrt{\beta a_d}
\]

(19-b)

\[
d = 0.96\beta^2 - 0.4\beta + 1.1 = 0.96(2a/b) + 0.4(2a/b) + 1.1
\]

(20)

\[
N_{p,s,d} = \int_{a_i}^{a_d} \frac{1}{C((\Delta K_{\text{deb}})^m - (\Delta K_{th})^m)da} + \int_{a_d}^{a_f} \frac{1}{C((d \times \Delta K_{\text{deb}})^m - (d \times \Delta K_{th})^m)da}
\]

(21)

Thus, after obtaining the material property \( \varepsilon_{a,max} \) from the manufacturer, Eq. (14) can be used to determine \( K_{\text{debond}} \), and Eq. (19-a) or Eq. (19-b) can then be used to solve for \( a_d \). Together with Eq. (20) for defining \( d \), Eq. (21) can be used to obtain \( N_{p,s,d} \). Based on previous studies \([6,15,35,49,50]\), for double-strengthened specimens with CFRP covering the initial crack, most results are better following this.
Thus, the modified Paris Law proposed in this paper was verified to be safer than the previous one.

3.5. A general computer program: “EasyFatigueforFSS”

A general computer program was created for designing strengthened steel members with initial fatigue cracks. Practical design should meet both the fatigue criterion and the strength criterion.

(1) Fatigue criterion

The fatigue criterion is found in Eqs. (19)–(21).

(2) Strength criterion

During fatigue loading, the crack length $2a$ develops from $2a_i$ to $2a_f$. The nominal stress shared by the steel under the static design force ($F_d$) is named $\sigma_d$ and can be obtained from Eq. (22-a) or Eq. (22-b). When the half crack length $a$ reaches its final value $a_f$, $\sigma_d$ reaches its maximum value $\sigma_{\text{max},d}$. $\sigma_{\text{max},d}$ should not surpass the design strength of the steel ($f_d$), which is the yield stress ($f_y$) divided by a safety factor. This is the strength criterion for a strengthened specimen. Based on this, $a_f$ can be obtained using Eq. (23), which safely promises that the crack propagation stops before the final fracture stage.

Table 2

<table>
<thead>
<tr>
<th>Material</th>
<th>Material properties</th>
<th>Geometric parameters (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>$E_s=201$ GPa</td>
<td>$l_s=400$</td>
</tr>
<tr>
<td></td>
<td>$\nu_s=0.33$</td>
<td>$b_s=70$</td>
</tr>
<tr>
<td></td>
<td>$t_s=10$</td>
<td>$t_s=0.94$</td>
</tr>
<tr>
<td></td>
<td>$b_s=70$</td>
<td></td>
</tr>
<tr>
<td>FRP composite consisting of FRP fiber and matched adhesive</td>
<td>$E_c$</td>
<td>$l_c=200$</td>
</tr>
<tr>
<td></td>
<td>$230$ GPa</td>
<td>$b_c=70$</td>
</tr>
<tr>
<td>FRP composite consisting of FRP fiber and matched adhesive</td>
<td>$16.69$ GPa</td>
<td>$t_c=0.94$</td>
</tr>
<tr>
<td></td>
<td>$G_{xy}$ direction:</td>
<td></td>
</tr>
<tr>
<td>FRP composite consisting of FRP fiber and matched adhesive</td>
<td>$8.57$ GPa</td>
<td>$G_{xy}=8.57$ GPa</td>
</tr>
<tr>
<td></td>
<td>$G_{xz}$ direction:</td>
<td></td>
</tr>
<tr>
<td>FRP composite consisting of FRP fiber and matched adhesive</td>
<td>$G_{xz}=8.57$ GPa</td>
<td>$G_{xz}=8.57$ GPa</td>
</tr>
<tr>
<td></td>
<td>$G_{yz}$ direction:</td>
<td></td>
</tr>
<tr>
<td>FRP composite consisting of FRP fiber and matched adhesive</td>
<td>$G_{yz}=0.91$ GPa</td>
<td>$G_{yz}=0.91$ GPa</td>
</tr>
<tr>
<td>Adhesive</td>
<td>$G_a=0.908$ GPa</td>
<td>$t_a=0.22$</td>
</tr>
<tr>
<td>Adhesive</td>
<td>$\nu_a=0.33$</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 8. Effects of the damage ratio, stiffness ratios (i.e., $S_i/S_s$ and $S_v/S_s$) and nominal stress on $d$. (a) Effect of the damage ratio on $d$. (b) Effect of $S_i/S_s$ on $d$. (c) Effect of $S_v/S_s$ on $d$. (d) Effect of nominal stress on $d$. 


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The above equations are for CFRP sheets. For CFRP laminate, $E_f$, $b_f$, and $t_f$ should be used in the same way as $E_c$, $b_c$, and $t_c$. Both the fatigue criterion and strength criterion ensure that the predicted fatigue life is available not only under fatigue loads but also under the design static force, which is defined as the available fatigue life $N_a$.

In this case, the program “EasyFatigueforFSS” is proposed for calculating the allowable stress or the available fatigue life of FRP strengthened steel members with an initial crack under fatigue. Thus, the design can be conducted oriented to the available fatigue life, and Fig. 10 shows the flow chart of the computer program “EasyFatigueforFSS”.

4. Design examples

Like any other design process, iteration may be required to reach the final design. Three major parameters can be changed in terms of CFRP patch design, namely the modulus of elasticity (i.e. type of CFRP), width of CFRP (i.e. the amount of CFRP) and thickness of CFRP (i.e. number of layers of CFRP). In the following examples, only the final design is shown.

4.1. Steel members without initial fatigue cracks

Example 1. In a truss, a butt-welded end-to-end connection between two circular hollow sections (CHSs) is subjected to variable amplitude tensile fatigue loading. The transverse butt weld splice is welded from one side with full penetration, and the root is inspected following the appropriate inspection procedure. The geometric parameters are shown in Table 3, and the expected stress ranges ($S_i$) and corresponding estimated numbers of cycles ($n_i$) are shown in Table 4. The detail is Fig. 9.

Comparison of the predicted fatigue lives (with debonding and without debonding) and experimental fatigue lives [6,15,35,49,50].

<table>
<thead>
<tr>
<th>Stress range number</th>
<th>Expected stress range ($S_i$)</th>
<th>Estimated number of cycles ($n_i$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>80</td>
<td>200,000</td>
</tr>
<tr>
<td>2</td>
<td>60</td>
<td>1,000,000</td>
</tr>
<tr>
<td>3</td>
<td>35</td>
<td>2,000,000</td>
</tr>
<tr>
<td>4</td>
<td>20</td>
<td>400,000</td>
</tr>
</tbody>
</table>

Table 4

Expected stress ranges and corresponding estimated number of cycles in Example 1.

<table>
<thead>
<tr>
<th>CHS outer diameter</th>
<th>CHS wall thickness</th>
<th>Elastic modulus of CFRP composite $E_c$</th>
<th>Wall thickness of CFRP composite made of CFRP sheets and matched adhesive</th>
</tr>
</thead>
<tbody>
<tr>
<td>101.6 mm</td>
<td>6.4 mm</td>
<td>150 GPa</td>
<td>3.0 mm</td>
</tr>
</tbody>
</table>

Table 3

Material and geometric parameters of the CHS in Example 1 and Example 2.

Fig. 10. Flow chart of the computer program “EasyFatigueforFSS”.

located on a non-redundant load path, and the consequence of failure is considered high.

The solution according to Eurocode 3 Part 1.9 is as follows:

**Step 1.** Determine the partial safety factor.
For a non-redundant load path and high consequence, $γ_M = 1.35$.

**Step 2.** Determine the appropriate detail category.
From Table 1, the detail category should be 71 for a butt-welded end-to-end connection of CHSs with $t < 8$ mm. Thus, $S_{DC} = 71$ MPa, $S_{CA} = 52$ MPa, and $S_{CO} = 29$ MPa.

**Step 3.** Check the stress range.
The given stress range No. 4 ($S_i = 20$ N/mm²) is less than $S_{CO}$. Therefore, this stress range is not included in the calculations below.

**Step 4.** Determine the fatigue strength $N_i$ at $S_i$ using Eq. (1):

$$
N_i = \frac{S_i}{\gamma_M} = \frac{20}{0.7} = 28.57 \text{ N/mm}^2.
$$

**Step 5.** Check the fatigue damage accumulation.
To check the fatigue damage accumulation, Eq. (5) can be cited:

$$
D = \sum \frac{n_i}{N_i} = \frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3} + \frac{n_4}{N_4} = \frac{30 \times 10^6}{1.38 \times 10^6} + \frac{100}{3.15 \times 10^6} + \frac{1000}{1.38 \times 10^6} + \frac{2000}{3.15 \times 10^6} = 0.46 \leq 1.0,
$$

not satisfactory.

The above solution is based on Eurocode 3 Part 1.9. If we apply AS 4100, the damage ratio is 1.64, which is not satisfactory either. The difference is due to the capacity factor $\phi = 0.7$ adopted in AS4100, which has an equivalent partial safety factor of approximately $1/0.7 = 1.43$ compared with that of 1.35 in Eurocode 3 Part 1.9.

Therefore, using CFRP sheets to strengthen the connection is recommended. The layout is shown in Fig. 11, and the material and geometric parameters are shown in Table 3. Then, the calculation for the reinforcement is conducted as follows:

**Steps 1–3 are the same as above.**

**Step 4.** To calculate the reduced stress range, Eq. (2-a) can be cited:

$$
N = S / \gamma_M = \frac{S_{Test}}{\gamma_M} = \frac{S_{Test}}{0.7}.
$$

**Step 5.** The solution according to Eurocode 3 Part 1.9 is as follows:

$$
D = \sum \frac{n_i}{N_i} = \frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3} + \frac{n_4}{N_4} = \frac{30 \times 10^6}{1.38 \times 10^6} + \frac{100}{3.15 \times 10^6} + \frac{1000}{3.15 \times 10^6} + \frac{2000}{3.15 \times 10^6} = 0.46 \leq 1.0,
$$

not satisfactory.

The expected stress ranges ($S_i$) and corresponding estimated numbers of cycles ($n_i$) are shown in Table 5. Assume that the detail is located on a non-redundant load path and that the consequence of failure is considered high.

(a) Determine the fatigue life of the detail.

**Solution:** Steps 1–4 are the same as in Example 1.

**Step 5.** Calculate the fatigue damage accumulation per day using Eq. (5):

$$
D = \sum \frac{n_i}{N_i} = \frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3} + \frac{n_4}{N_4} = \frac{30 \times 10^6}{1.38 \times 10^6} + \frac{100}{3.15 \times 10^6} + \frac{1000}{3.15 \times 10^6} + \frac{2000}{3.15 \times 10^6} = 1.53 \times 10^{-4}.
$$

Therefore, the connection can bear $1/(1.53 \times 10^{-4})/365 = 17.9$ years of fatigue loading.

(a) The fatigue life needs to be increased by 25%, and CFRP strengthening is considered one of the options. If the reinforcement layout is the same as in Example 1 (as shown in Fig. 11) and the available CFRP sheet has a thickness of 0.176 mm, is one layer of CFRP...
strengthening sufficient? If not, how many layers of CFRP are required? The Young's modulus of the steel ($E_s$) is 204 GPa, and that of the CFRP sheet ($E_f$) is 230 GPa.

**Solution.** For a single-sided repair, when we do not know the amount of adhesive, Eq. (3-a) can be cited. If we apply one layer of CFRP, the relationship of $S$ with CFRP, $i$ and $S$ without CFRP, $i$ can be obtained:

\[
S_{\text{with CFRP}i} = S_{\text{without CFRP}i} \times \frac{1}{1 + \left(\frac{230}{204}\right) \times (1) \times \left(\frac{0.176}{6.4}\right)}
\]

Applying Eq. (1),

- $N_1 = (52.6)^3 \times 2 \times 10^6 / 77.6 = 0.62 \times 10^6$
- $N_2 = (52.6)^3 \times 2 \times 10^6 / 58.2 = 1.48 \times 10^6$
- $N_3 = (52.6)^3 \times 5 \times 10^6 / 34.0 = 44.31 \times 10^6$

Fig. 13. Fatigue cycle vs. half crack length curves, diagram of crack propagation and debonding propagation, and variation of $\sigma_d$ at different crack lengths (a) figures in the crack developing procedure for Example 3 [51]. (b) figures when the crack reaches the final crack length for Example 3 [51]. (c) Figures in the crack developing procedure for Example 4 [35]. (d) Figures when the crack reaches the final crack length for Example 4 [35].
Step 5. Check the fatigue damage accumulation using Eq. (5):

\[ D = \sum_{i=1}^{n} \frac{n_i}{N_i} = \frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3} = \frac{30}{6.4 \times 10^5} + \frac{100}{1.48 \times 10^8} + \frac{1000}{44.51 \times 10^5} = 1.39 \times 10^{-4}. \]

Therefore, the connection can bear \( \frac{1}{1.39 \times 10^{-4}} / 365 = 19.7 \) years of fatigue loading, which is not sufficient. If three layers of CFRP are applied, then

\[ S_{\text{with CFRP},i} = S_{\text{without CFRP},i} \times \frac{1}{1 + \left( \frac{230}{204} \right) \times \left( \frac{0.176}{6.4} \right) \times (1 + 0.78 + 0.56)} = S_{\text{without CFRP},i} \times 0.93 \]

Applying Eq. (1),

\[ N_1 = (52.6)^3 \times 2 \times 10^6 / 74.4^3 = 0.71 \times 10^6 \]
\[ N_2 = (52.6)^3 \times 2 \times 10^6 / 55.8^3 = 1.68 \times 10^6 \]
\[ N_2 = (52.6)^2 \times 5 \times 10^6 / 32.6^2 = 54.68 \times 10^6 \]

**Step 5.** Check the fatigue damage accumulation using Eq. (5):

\[ D = \frac{1}{N} \sum_{i=1}^{n} \left( \frac{N_i}{N} \right) \right] = \frac{30}{0.71 \times 10^6} + \frac{100}{1.68 \times 10^6} + \frac{1000}{54.68 \times 10^6} = 1.20 \times 10^{-4}. \]

Therefore, the connection can bear \(1/(1.20 \times 10^{-4})/365 = 22.8 \) years of fatigue loading, which is sufficient.

### 4.2. Steel members with initial fatigue cracks

The following results are from the program “EasyFatigueforFSS” on the MATLAB platform, as described in Section 3.5. For the following two examples, the specimens are shown in Fig. 12(a) and (b). In every sub-figure of Fig. 13, the fatigue life vs. half crack length curves for the pure steel beam, the strengthened steel beam considering debonding, and the strengthened steel beam without considering debonding are shown on the left side. The diagram of the crack propagation and debonding propagation of the strengthened steel beam considering debonding is shown in the top right corner, and the variation of \( a_f \) due to crack growth is shown in the bottom right corner.

#### Example 3.

This example is based on a steel beam (Specimen B5 in [51]) tested in a previous study; it was a standard hot-rolled H350 \( \times 150 \times 7 \times 11 \) steel beam. To simulate actual damage, two U-shaped notches with a width of 8 mm and a length of 21.8 mm were cut on both sides of the tension flange at the midspan. Fiber-reinforced composite plates were affixed with epoxy to the strengthened locations, which was a single-side reinforcement. The fatigue test was conducted under a constant amplitude cyclic load [51]. The related parameters are as follows: \( \Delta \sigma = 101.5 \) MPa, \( E_i = 291,279 \) MPa, \( b_i = 175 \) mm, \( b_2 = 150 \) mm, \( E_2 = 199,000 \) MPa, \( t_m = 11 \) mm, \( t_s = 3 \) mm, \( a_s = 21.8 \) mm, \( C = 6.77 \times 10^{-13}, m = 2.88, R = 0.2, f_s = 330 \) MPa, \( f_d = 275 \) MPa, and \( e_{a_{\text{max}}} = 1.5 \% [51, 52] \).

To determine the available life, \( F_d = 400 \) kN. Without CFRP, the available life is 253,000; with CFRP, the available life considering debonding is 598,000, and that without considering debonding is 744,000. The experimental fatigue life of this specimen is 820,000, which is greater than the predicted available lives (i.e., 598,000 and 744,000).

#### Example 4.

This example is based on another steel beam called BD85d in Ref. [35]; it was a hot-rolled H194 \( \times 150 \times 6 \times 9 \) steel beam. A 15 mm-length crack was created by pre-fatigue loading on one side of the tension flange at the midspan. The two sides of the tension flange were reinforced with CFRP plates. The fatigue test was also conducted under a constant amplitude cyclic load [35]. The related parameters are as follows: \( \Delta \sigma = 85 \) MPa, \( E_i = 320,000 \) MPa, \( b_i = 150 \) mm, \( b_2 = 50 \) mm, \( E_2 = 206,000 \) MPa, \( t_m = 9 \) mm, \( t_s = 2.8 \) mm, \( a_s = 15 \) mm, \( C = 6.77 \times 10^{-13}, m = 2.88, R = 0.2, f_s = 435 \) MPa, \( f_d = 362.5 \) MPa, and \( e_{a_{\text{max}}} = 1.5 \% \), which is the same as in Example 3.

To determine the available life, \( F_d = 10 \) kN. Without CFRP, the available life is 562,000; with CFRP, the available life considering debonding is 818,000, and that without considering debonding is 1,101,000. The experimental fatigue life of this specimen is 1,680,000, which is greater than the predicted available lives (i.e., 818,000 and 1,101,000).

As shown in the above two examples, the predicted available lives are smaller than the experimental fatigue lives. It is seen that the consideration of the debonding and the strength criterion makes this design safe.

### 5. Conclusions

(1) The classification method can be used for reinforced steel members with CFRP if no initial fatigue cracks exist. When proper surface treatment is performed, the reduced nominal stress range due to the CFRP can be obtained using Eq. (2) or Eq. (3). Then, the fatigue life and accumulated damage can be obtained to determine whether the reinforcement is satisfactory.

(2) CFRP is shown to be effective for strengthening steel structures under fatigue. CFRP can extend the fatigue life under a certain loading condition or increase the allowable stress range when a certain fatigue life is desired.

(3) Based on previous studies and FEM, the interaction of debonding and crack growth in cracked steel plates strengthened with CFRP was studied. A full range of analysis was conducted for the crack propagation and development of the debonding area. A new modified prediction method for the fatigue life was proposed by considering the influence of debonding, and better results were obtained in most cases.

(4) An available life \( N_f \) is proposed for cracked steel members that can meet both the fatigue design standard and strength design standard. Design oriented to either the available life or the allowable stress can be conducted using the “EasyFatigueforFSS” program, making the design convenient and available to engineers.

It should be noted that the design method proposed in this paper is limited to steel plates or members in tension. More work is needed to extend the design to other loading cases such as bending and torsion and to welded connections.

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### References


