A Large-span Woven Web Suspension Roof System Made of High-strength FRP

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Summary

FRP WWS (fiber reinforced polymer woven web structure), a large-span roof system is presented in this paper firstly. It is composed of light-weight, high-strength and anticorrosion FRP strips, which are crossed and woven. The edge of the woven FRP web is fastened and anchored on an outer ring beam, and the tendons on the inner ring beam are prestressed in the out of plane direction which is to establish the geometric stiffness to bear various loads. The same effect can also be achieved by using some special heavy mass or roof suspended in the center. In the paper, the basic configuration and the construction scheme for a simple FRP WWS is presented. Three types of the geometric woven plane patterns are summarized and the geometric expression of the arbitrary weave web is introduced. Two approaches are proposed to determine the web's configuration.

Keywords: Large-span structure; suspension structure; fiber reinforced polymer, FRP; weave pattern; release minimum distance algorithms, RMDA; genetic algorithms, GA; objective function; geometric stiffness; segment intersection.

1. Introduction

With the growing economical and cultural needs, people are seeking to cover larger spaces than ever before. To achieve a large-span structure is also a long-time purchased aim for structure engineers. Recently, many large-span structures have been constructed in China, such as the National Grand Theatre, the Beijing Olympic Sports Center, the Guangzhou Olympic Sports Center and the Shenzhen Citizen Center. Nowadays, the most common structural materials used for large-span structures are concrete, steel, and composite membrane materials. Many of their corresponding structure forms, include thin shells, latticed frames, latticed shells, space trusses, cable suspended structures, membrane and cable-membrane structures, tensegrity structures, and so on[1]. However, the self-weight of steel and concrete is higher, while their strength is relatively lower, which limited their use in larger spans. Moreover, the potential hazard and long-term expensive maintenance cost, all of which are resulted by the corrosion of steel member and steel reinforcement, have to be considered. Hence, in order to achieve larger spans, there must be some break-through in structural types and structural materials.

FRP is a new kind of structural material used in civil engineering recently. Due to its favourable properties like corrosion resistance, high strength, low weight, good fatigue performance, and low maintenance cost, it is considered as an ideal material to build large-span structures. At present, the researchers are trying to make rational structure types with FRP in order to achieve larger span than using tradional materials. For example, Ken-ichi Maeda et al. have conceived a 5000 meter-span suspension bridge using FRP [2].

FRP woven web structure presented in this paper, is one proposal to ultilize the FRP characteristic in the large-span roof system. In FRP WWS, the high-strength FRP strips are crossed and woven like the bamboo strips in Chinese bamboo mat to form a web in plane. The outer edge of the web is fastened and anchored on an outer ring beam, and an inner ring beam is installed to anchor the FRP strips at the center of the web, a small size FRP WWS model is shown in Fig 1. The FRP strips are prestressed at the primary level to make them straight during weaving construction. Then, the FRP web is tensioned by a displacement in the out of plane direction of the inner ring which is applied by a set of prestressed tendons or some especially suspended heavy mass from the inner ring beam. Thus, a tensioned FRP web, which can provide enough rigidity to afford the varied working load due to the geometric stiffness, forms a large-span roof system.

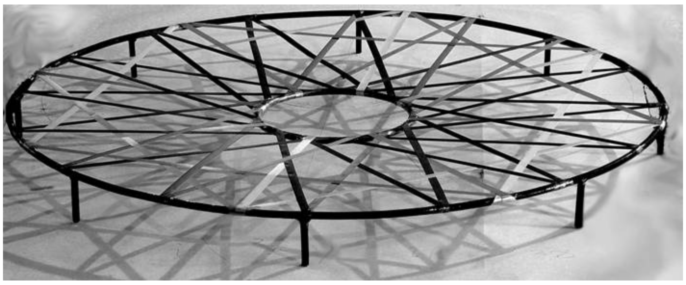

Fig. 1 FRP WWS model

The FRP WWS is almost analogous with the cable network structure and the cable-membrane structure: their members are flexible; and the geometric stiffness resulted by tension is utilized to resist the loads. However, the FRP WWS have its unique advantages: (1) FRP strip is suited in super large-span structures due to its low self-weight and high tensile strength lengthwise, its higher strength and modulus in lengthwise can be utilized efficiently while the lower in the crosswise can be avoided because all strips are tensioned in the FRP WWS; (2) an additional damping can be provided by the friction between the crossed FRP strips, which may decrease the vibration response of the structure under the winding load or the earthquake load; (3) good architecture effects of the roofs can be attained as the strips are arrayed regularly and geometrically; (4) lower maintenance cost for the anticorrosive FRP and easy installation because of its light-weight.

In the paper, the basic configuration and the construction scheme for a FRP WWS is presented in detail. The geometric weaving patterns are summarized into three types, and the various types of the web structure shapes which can be used in actual structures are derived. The weaving pattern of the structures are defined in geometry and expressed by the vectors and matrices. Two geometric analysis approaches are presented based on the release minimum distance algorithms (RMDA) and the genetic algorithms (GA).

2. Configuration and Construction of FRP WWS

A simplest FRP woven web structure is composed of a FRP woven web, outer anchor ring beam, inner anchor ring beam, and an additional weight or prestressed tendons, as shown in Fig.1. The web is woven with FRP strips, CFRP strips or other high-performance hybrid FRP strips are suggested. CFRP strip is manufactured in pultrusion in general by carbon fiber and resin matrix with a fiber volume ratio about 65%. In the lengthwise direction, the elastic modulus is more than 150GPa, the strength is above 2400MPa, and the coefficient of thermal expansion is about 0.2×10^{-6}℃. The strips can be curved and circumvoluted as Fig.2. A typical CFRP strip is able to resist more than 400kN load in tension while the weight of 300m long strip is less than 70kg. In comparison, the self weight of 300m long high strength steel cable which can resist same load is more than 500kg.

Fig. 2 CFRP strips

The strips are arrayed in plane by some definite rules. The weave repeats after a certain number of strips. Each strip passing over one crossing strip and under the next forming the web like fabric. A partial web is shown in Fig.3. Actually, the number of the crossed strips and the angle of strips are the varied parameters, two crossed strips at 90° to each other are shown in Fig.4(a), three crossed strips at 60°shown in Fig.4(b), and four strips at

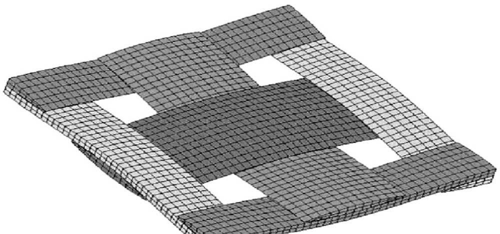

Fig. 3 Part of woven web[3]

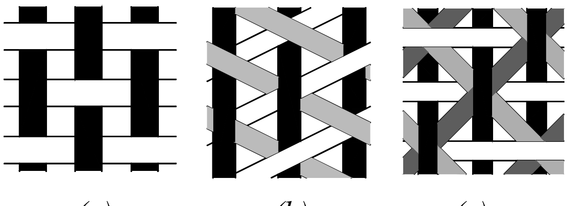

Fig. 4 Weave styles

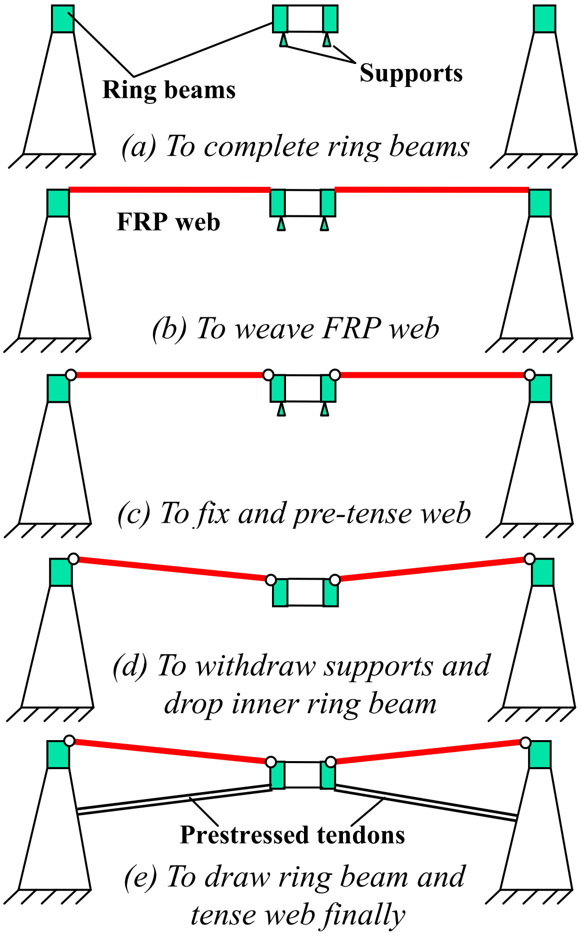

Fig. 5 Construction steps

45° shown in Fig.4 (c). On the cross points, strips can either be fixed and clamped altogether with bond or a proper release may be provided rather than full fixity. Thus, crushing and the sliding friction will exist at the crossing. The static friction between strips provides the stiffness under the static load while the slipped friction exhausts the kinetic energy of structure under the dynamic load condition.

The edge of woven FRP web is fixed on the ring beams and each strip is stretched within some initial tension. Hinge joints are installed between the ends of strips and the ring beams. Then, following the construction steps as shown in Fig.5, the simplest FRP WWS can be completed. The outer ring beams and the inner one on the temporary supports should be completed firstly. The outer ring beam will be in compression and the inner one will be in tension. Then, the woven web will be pre-tensed and fixed in-plane with weaving the strips one by one following definite weaving style. Withdrawing the supports and drawing the ring beam in the out of plane direction by the self-weight of the inner ring beam and the prestressed tendons would be the last step. According to the force decomposition, a small the out of plane forced can lead into large in-plane components forces and make the web tensioned. Thus, the web can be provided with sufficient geometric stiffness and can resist the various loads.

3. Weave Patterns in Plane

The FRP web is the basic component in the WWS system. Various weave patterns can be adopted in plane, which may result in different mechanical behaviors. So the weave pattern analysis is the primary task before the structure analysis for FRP WWS. Each weave pattern is composed of the regular arrayed straight line segments to represent the FRP strips, and the outer and inner boundaries to represent the ring beams on which the ends of the line segments are located. The laying relations between the strips at the intersection will be considered in the next section. Thus, the pattern is the plane geometry by segments and boundaries. Various patterns can be summarized in the three types as following: tiled type, radiated type and inscribed type.

3.1 Tiled Type

The web weave that is independent of the boundary is defined as tiled type pattern. A normal plain weave web with circular boundary is shown in Fig.6. The outside of the outer boundary and the inside of the inner are cut off while the web is uniform, which belongs to the tiled type like a piece of cloth tiling over the ring beams. The web of this type can be considered as a uniform fabric to analyze.

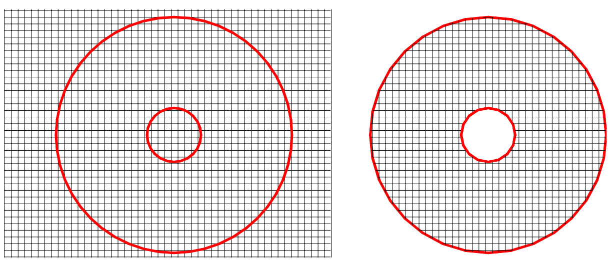

Fig. 6 Tiled type weave

3.2 Radiated Type

For radiated type, two ends of each FRP strips are

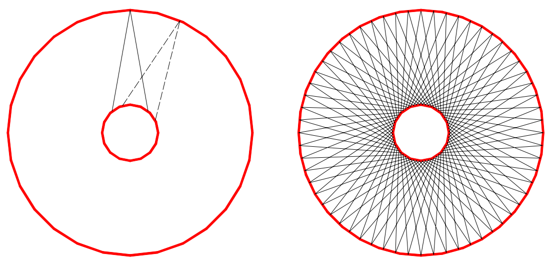

Fig. 7 Radiated type weave

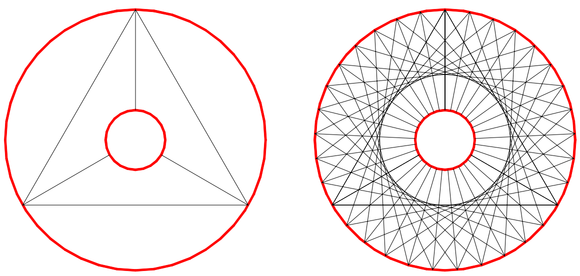

Fig.8 Inscribed type weave

located on the inner and the outer ring consecutively, and the web is made up of number of repeated sets each of which is composed of certain number of segments. Thus, the rule between sets and segments in a set are the basic principle to form the web. For example, the web shown in Fig. 7 is a radiated type weave web. Its rule in set is that two segments corner at 20° and their ends on the outer ring coincide, and then they are copied after every 6° round the center which is the rule between sets. Each set can be analyzed as a basic unit in the structure. The strips in radiated type can be utilized sufficiently.

3.3 Inscribed Type

In an inscribed type weave, the inscribed polygon of the outer boundary and the segments from the polygon's corners to the center compose the basic set; and the set is copied according to the rule between sets. Fig.8 shows an inscribed type weave formed by an inscribed triangle. In this type, some dense zone can be formed which can improve the local behavior of the web.

These three types of weave pattern are defined in order to describe the various weave patterns. Actually, they should be used together and laid up in a FRP WWS.

4. Geometric Analysis Approaches of Weaving Patterns

All these types have a definite arraying principle in plane, but the actual FRP WWS is a 3D structure. The laying of the strips on each intersection should be determined to make the configuration of the assembled strips well, which cannot be acquired by planar geometry. The configuration of the web should be expressed in mathematic, analyzed and optimized to determine the weave detail of each strip.

4.1 Expressions

The ring beams anchoring the FRP web can be abstracted as the boundary of one field in plane, and the strips can be regarded as segments with two extreme points in this field. All of the extreme points locate on the boundary. The plane is defined as ***X-Y*** and the boundary can be expressed as a function:

$$b(x,y)=0 \tag{1}$$

If there are n line segments in the field which are numbered from 1 to n, the start point of the ith line, $1 \leqslant i \leqslant n$, is noted as $(x_{i\mathrm{S}},y_{i\mathrm{S}})$, and the end is $(x_{i\mathrm{E}},y_{i\mathrm{E}})$. They are the coordinate values in Cartesian plane and must satisfy

$$b(x_{i\mathrm{S}},y_{i\mathrm{S}})=0, b(x_{i\mathrm{E}},y_{i\mathrm{E}})=0 \tag{2}$$

As two lines can only be intersected once in one plane, the total maximum intersections number is $n(n-1)/2$. The intersection of the ith line and the jth line is noted as (x_{ij},y_{ij}) which is its coordinate value in plane also, hence

$$x_{ij}=x_{ji}, y_{ij}=y_{ji} \tag{3}$$

If they don't cross in the field or $i=j$, then

$$x_{ij}=x_{ji}=\infty, y_{ij}=y_{ji}=\infty \tag{4}$$

Thus, the coordinate value of the intersections can be assembled and expressed into two symmetric matrices:

$$X=\begin{bmatrix}\infty & x_{12} & \cdots & x_{1j} & \cdots & x_{1n}\\ x_{21} & \infty & \cdots & x_{2j} & \cdots & x_{2n}\\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots\\ x_{i1} & x_{i2} & \cdots & x_{ij} & \cdots & x_{in}\\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots\\ x_{n1} & x_{n2} & \cdots & x_{nj} & \cdots & \infty\end{bmatrix}\qquad Y=\begin{bmatrix}\infty & y_{12} & \cdots & y_{1j} & \cdots & y_{1n}\\ y_{21} & \infty & \cdots & y_{2j} & \cdots & y_{2n}\\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots\\ y_{i1} & y_{i2} & \cdots & y_{ij} & \cdots & y_{in}\\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots\\ y_{n1} & y_{n2} & \cdots & y_{nj} & \cdots & \infty\end{bmatrix}\tag{5}$$

They can be determined from $(x_{iS},y_{iS}),(x_{iE},y_{iE})$ (where i=1,2,…,n) and Eq.1 by calculating the intersections coordinates in analytical geometry. A simple example is shown in Fig.9 where there are four lines shown. However, the laying relations on the intersections can not been determined directly. From Fig.6, 7 and 8, it cannot be seen whether a line lies above another one or below on the intersection. Thus, a parameter is needed to describe the laying relations on the intersection, which is defined as z_{ij}. If $i<j$ and the ith line under the jth line, then $z_{ij}=1$, $z_{ji}=-1$; if $i<j$ and the ith line over the jth line, then $z_{ij}=-1$, $z_{ji}=1$; if they are not crossed or $i=j$, then $z_{ij}=0$, $z_{ji}=0$. For example, a web configuration as shown in Fig.9 can be expressed as

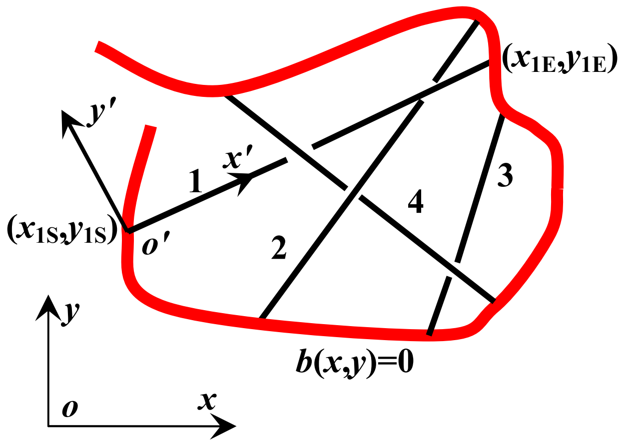

Fig.9 Segments in Field

$$Z=\begin{bmatrix}0 & -1 & 0 & 1\\ 1 & 0 & 0 & -1\\ 0 & 0 & 0 & 1\\ -1 & 1 & -1 & 0\end{bmatrix}\tag{6}$$

If there are more than two lines crossing in one point, a similar definition for $\boldsymbol{Z}$ can be expanded but it is more complex than two lines. In this paper, the intersection by only two lines is considered.

4.2 Solution and Optimization

For a certain weave pattern, the best configuration is regarded as that each strip must pass over a crossing one and under the next alternatively one by one, like the lines in Fig.9. Thus, this relation can be expressed for any two adjacent intersections (x_{ip},y_{ip}) and (x_{iq},y_{iq}):

$$z_{ip}+z_{iq}=0\tag{7}$$

This condition is called the completed configuration for the ith line segment. If all of the segments in the web are in completed configuration, the web is called the completed web. Actually, it may not be possible for every segment in a web, so two approaches are presented in the following to get the solution of $\boldsymbol{Z}$ which configures the web as completed as possible. Based on these two approaches, the computer program can be developed.

4.2.1 Release Minimum Distance Algorithms (RMDA)

RMDA is an ergodic method. The procedures are as following:

- Step 1: coordinate transformation. Move the origin of the plane coordination system to the start point of the line 1,where the line 1 is x' axis, as shown in Fig.9; and $\boldsymbol{X}\rightarrow\boldsymbol{X'}$, $\boldsymbol{Y}\rightarrow\boldsymbol{Y'}$. The first row of $\boldsymbol{Y'}$, $\{y_{1i}'\}=0$ where(i=1,2,…,n).
- Step 2: sequence ordering. Order the first row of $\boldsymbol{X'}$, from small to large, the infinitudes will be located in the rear half; define a vector $\{s_1\}$ to record the sequence of i in the ordered $\{x_{1i}'\}$, let $s_{1,m+1}=1$ on the first position of all infinitudes, where m is the number of the segments crossed by line 1.
- Step 3: initial solution: determine $\{z_{1i}'\}$, which is $\{z_{1i}\}$ ordered by $\{s_1\}$, as 1 and -1 changed one another from the beginning to s_{1m}, and the value as 0 from $s_{1,m+1}$. Thus, $\{z_{1i}\}$ is acquired, which ensures the line 1 is in the completed configuration.
- Step 4: the next segment is the jth line, where $j=s_{11}$. Then, repeat Step 1 to 2 by only replacing

line 1 as the jth line.

- Step 5: the solution of $\{z_{ji}'\}$ is determined as 1 and -1 changed one another from the beginning to s_{jm} while $z_{j1}+z_{1j}=0$ must be satisfied, which can be acquired by changing the value of the first number in $\{z_{ji}'\}$.
- Step 6: the next segment is the kth line, where $k=s_{j1}$. Then, repeat Step 5 where replace j as k. If any $\{z_{ki}'\}$ can ensure the kth line is in the completed configuration, then repeat this step, otherwise go to the next step.
- Step 7: find the adjacent intersections with the minimum distance on the kth line, assume they are p and q, and release the constrained condition of $z_{kp}+z_{kq}=0$. Let $z_{kp}=z_{kq}$, then $\{z_{ki}\}$ can be constructed, which ensures the kth line is in the completed configuration except for p and q.

Following these steps, a ***Z*** matrix can be acquired. Actually, ***Z*** of a completed web can be determined by this approach without releasing the minimum distance.

4.2.2 Genetic Algorithms (GA)

GA is an optimization approach. The matrix of ***Z***, the initial solution of which is generated randomly, is optimized by GA function to minimize the objective function of the system. For this problem, the objective function is

$$\min(C) = \min \sum_{i=1}^{n} \sum_{j=1}^{n} [(z_{ij} + z_{ik})^2 \cdot d_{jk}] \tag{8}$$

where the intersection between the ith line and the jth is adjacent the intersection between the ith line and the kth, the d_{jk} is the distance between these two points. There is a lot of research available about GA for minimizing a function. This approach is a directly search method due to ***Z*** has been known in every step. The GA function will optimize ***Z*** continually until function C is small enough.

5. Conclusion

The FRP WWS is presented in this paper. The configuration and the construction scheme are introduced in detail. The woven patterns are summarized and analyzed. Key points as following:

- The FRP WWS is composed of three parts: FRP woven web, ring beams and the out of plane tension system. There are five steps to construct a simple FRP WWS.
- Various weave patterns can be summarized in three types: tiled type, radiated type and inscribed type.
- The woven web configuration can be expressed by there matrices: ***X***, ***Y*** and ***Z***. ***X*** and ***Y*** are the coordinate matrices of the intersections in plane, which are determined by analytical geometry. ***Z*** is the location matrix of the intersections, which is determined by RMDA or GA.

6. Acknowledgement

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