REVIEW AND NEW PROPOSALS FOR PERFORMANCE INDICES OF FLEXURAL MEMBERS*

Peng Feng, Lieping Ye, Heng Zhao and Jiangbo Zhuang

Department of Civil Engineering, Tsinghua University
Beijing, 100084, P.R. China

ABSTRACT: With the development of new structural materials including FRP (fiber reinforced polymer), glass, stainless steel, the performance of structures have a great change from the steel and the steel reinforced concrete structures. The conventional ductility index that describes the inelastic deformation capacity for the traditional materials cannot longer suit the structures with new materials. A new term, "deformability", has been proposed to extend the concept of the ductility. In this paper, more than ten existing definitions of performance indices for the flexural members are reviewed. A new concept that is design-aimed state is defined relative to ultimate failure state after carrying out analysis based on the comparison in term of concepts and methodologies of the reviewed indices. A new approach to achieve the uniform performance indices and to determine the design-aimed state is proposed also. A set of uniform performance indices, including the deformability index *D*, the strength index *S*, the deformation energy index *Y* and the overall performance index *F* are presented. The moment-curvature curve of five type of flexural members, including steel beam, steel-reinforced concrete beam, prestressed concrete beam, FRP bars reinforced concrete beam and all-FRP beam, are analyzed using the suggested indices. *F*-factor is suggested to determine the design-aimed point for all flexural members.
KEYWORDS: ductility; deformability; overall performance index; FRP; beam; design-aimed state

1. INTRODUCTION

The bearing capacity and the deformable capacity are the two key performances considered in design of the civil structures. In the design approaches based on the elastic analysis like the allowable stress method, bearing capacity index is mainly considered in comparison with deformation. With the application of steel and reinforced concrete and with the study on their elasto-plastic behaviors, ductility was presented in almost all of the recent design approaches. The ductility is a measure of the ability of a material, section, structure element, or structural system to sustain inelastic deformation prior to collapse, without significant loss in resistance (Naaman 1986). The mild steel and the ordinary concrete, which are used widely in the recent eighty years, are regarded as the good ductile structural materials because they can provide obvious inelastic deformation. The structures composed of them can absorb much energy by plastic deformation and can warn about collapse by obvious deformation. Thus, the ductility is the paramount safety characteristic and the performance index besides the bearing capacity of the structures built with the common materials.

Recently, some new structural materials such as high-strength steel, fiber reinforced polymer (FRP) and structural glass are applied in civil structures. Their mechanical behaviors are much different from the mild steel and the ordinary concrete: (i) there is a yield plateau or a descending part in the stress-strain curves of the conventional materials while the curves of the news materials always ascend before break; (ii) the former show more obvious residual deformation after unloading from the plastic part as compared to later; (iii)in the ultimate deformation plastic carries the major share as compared to elastic deformation in former, however, a very minor share in later. The respective differences also exist between the load-deflection curves of the structures composed of them. Fig.1 (a), (b) and (c) show the load-deflection curves of SPL, an all-FRP deck, B9, a reinforced concrete beam strengthen with FRP sheet (Ye 2003) and A80-80-s, a FRP prestressed concrete beam (Zou 2003) respectively. They may be regarded as no ductility according to the conventional concepts because of no yield plateau or

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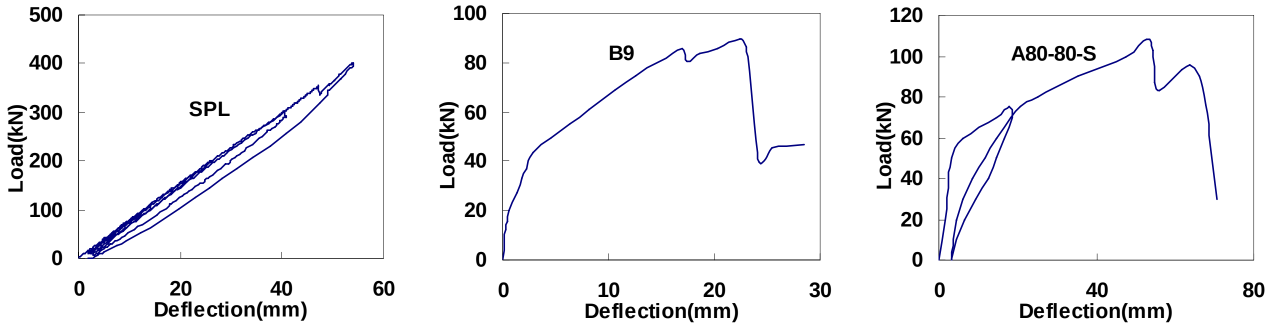
descending part before their failure. Thus, a wrong conclusion that these elements cannot be used in structures may be drawn. Actually, they can absorb much energy and can warn before failure by undergoing significant elastic deformation as the deck shown in Fig.2 whose load-deflection curve is shown in Fig.1(a). Therefore, the conventional ductility index is no longer suitable for the members like it.



(a) (b) (c)

Figure 1 Load-deflection curves of the flexural members with new materials

Figure 2 Test of an all FRP deck

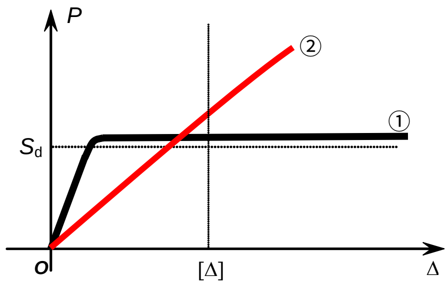


Figure 3 Two different load-deflection relations

On the other hand, the strength and the ductility, which are the conventional performance indices, cannot scale the elements with various behaviors more comprehensively on same design conditions. The load-deflection relations of two different types of members are shown in Fig.3, where S_d is the required load capacity and [Δ] is the allowed deformation limit for design. Although they both satisfy the work requirements, ① is better than ② in terms of the ductility while ② have larger strength safety margin than ①. It cannot be determined which one can provide more safety only by strength and ductility. So the new uniform indices are needed to characterize the overall performance of structures, especially for the elements with new structural materials.

In the recent studies on the new material elements, several modified ductility indices were defined, and a new term "deformability", which means the deformable capacity including the elastic and the inelastic, was presented. However, most of them were proposed in terms of only one type of members. In this paper, the existing performance indices of the flexural members are reviewed and summarized. The concepts and the methodologies for defining the performance indices are discussed. Based on the discussion, the four performance indices are proposed, including the deformability index *D*, the strength index *S*, the deformation energy index *Y* and the overall performance index *F*. The moment-curvature relations of five types of members are analyzed using these indices from different viewpoints. The overall performance is evaluated with the proposed indices for design.

2. REVIEW OF THE EXISTING PERFORMANCE INDECIES

2.1 Conventional Ductility Indices

The conventional ductility index was presented based on the reinforced concrete (RC) and the steel members. Because their load-deflection relations may be modeled as the ideal elasto-plastic curve with a yield point approximatively, as shown in Fig.4, the yield point as a key point is used in design. The yield load as the design load is predicted and the plastic deformation from yield to failure acts as the safety margin. Therefore, the yield point and the ultimate failure point are considered in the definition of the conventional ductility indices as following.

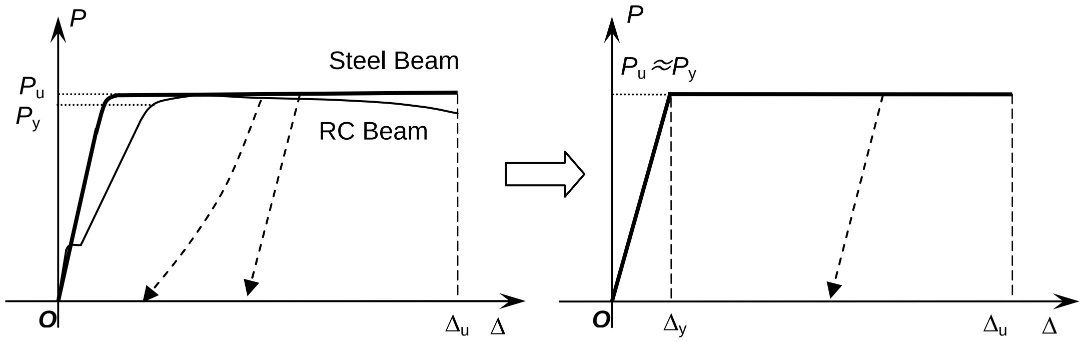


Figure 4 Definition of conventional ductility index

Conventional deformation ductility index is the ratio of the ultimate deformation over the yield deformation, which can also be defined as the curvature ductility index, the rotational ductility index and the displacement (or deflection) ductility index in term of the variables used.

$$\mu_\Phi = \frac{\Phi_u}{\Phi_y}, \quad \mu_\theta = \frac{\theta_u}{\theta_y}, \quad \mu_\Delta = \frac{\Delta_u}{\Delta_y} \tag{1}$$

For flexural elements, the curvature relates to the section of members, the rotation generally relates to the deformation of a part of the member and the deflection relates to the entire element. The deflection ductility is mostly used in research because it can be measured in test more easily than curvature and rotation. However, sometimes it is depended on the support conditions and the length of the members. Furthermore, it is difficult to determine the deflection of a member in plastic state. Actually, the moment-curvature relation is more comparable in various structures.

The energy ductility index is the ratio of total deformable energy at the ultimate limit state over the deformable energy on the yield point, that is

$$\mu_E = \frac{E_u}{E_y} \tag{2}$$

The energy ductility index that is based on more essential concept can describe more complex behavior of the structure though the yield point is still necessary for it. For the ideal elasto-plastic model composed of two straight lines, there is the relation

$$\mu_E = 2\mu_\Delta - 1 \tag{3}$$

2.2 Modified Ductility Indices and Deformability Indices

Because of the non-yield behavior in some structures, the conventional ductility indices have limitations in application, hence the modified ductility indices as following were proposed for them.

Ductility Index Proposed by Abdelrahman et al.

In 1995, a ductility model for beams prestressed by FRP tendons was presented by Abdelrahman et al (1995), which is based on the experimental research of four pretensioned concrete bridge T-beams. It was expressed in terms of section properties or member deformations as follows

$$\mu_{\Phi,A} = \frac{\Phi_u}{\Phi_l}, \quad \mu_{\Delta,A} = \frac{\Delta_u}{\Delta_l} \tag{4}$$

Φ_u and Δ_u are the curvature and the deformation at failure, while Φ_l and Δ_l are the equivalent curvature and deformation of the uncracked section at a load equal to the ultimate load, as illustrated in Fig.5. In their test, $\mu_{\Phi,A}$ are 6.3 and 8.2 for the beams prestressed by FRP and steel respectively while $\mu_{\Delta,A}$ are 5.5 and 5.2, respectively. The index is similar to the definition of Thompson and Park (1980). Essentially, a nominal yield point is defined for the prestressed beams in the model. Because the point is only related to the initial stiffness and independent of the rear part, it cannot reflect the overall performance of the beams and cannot be extended to the other types of members. Actually, several definitions for the nominal yield point for the members without obvious yield point have been proposed, such as Park's method (1983), the equivalent energy method (Guo 1999) and the geometric construction method (Guo 1999). However, the same defect limits their usage widely.

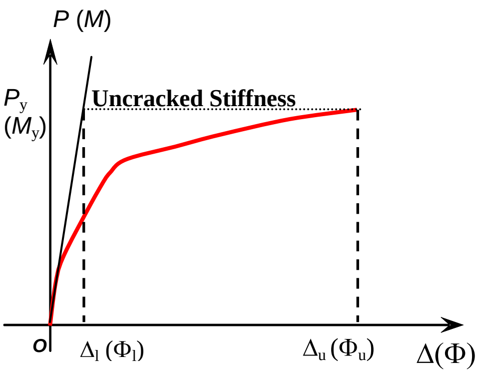


Figure 5 Abdelrahman et al's definition

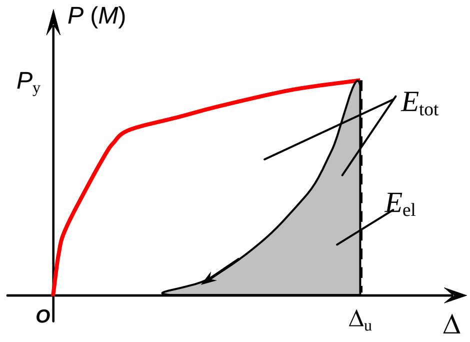


Figure 6 Naaman and Jeong's definition

Ductility Index Proposed by Naaman and Jeong

In 1995, a new definition of ductility index in form of energy was presented by Naaman and Jeong (1995). Twenty-four prestressed concrete beams by steel or FRP are computed using this index:

$$\mu_{\mathrm{E,N}} = \frac{1}{2}\left(\frac{E_{\mathrm{tot}}}{E_{\mathrm{el}}} + 1\right) \tag{5}$$

where E_{tot} is the total deformation energy, which is total area under load-deformation curve as shown in Fig.6, and E_{el} is elastic deformation energy, the area of the shadowed part in Fig.6. In the paper, the approach to determine unloading stiffness of the prestressed concrete beams is presented also. This index, which is applied for the beams without obvious yield point, has wider range for application than the convenient ductility indices. For the bilinear ideal elasto-plastic load-deformation curve, Eqn.1 and Eqn.5 are equivalent. However, it is difficult for some type of elements to determine the unloading stiffness accurately. Essentially, it still depends on the plastic deformation capacity of the beams. So it is unavailable for the members with the elastic materials like the one shown in Fig.1(a) though it is functional to the prestressed concrete beams.

Deformability and J-factor Presented by Mufti et al

Mufti et al (1996) presented a new concept of deformability versus ductility based on the research on FRP and steel reinforced concrete beams in 1996, and a set of performance indices as following is proposed,

strength index $$S_J = \frac{M_{\mathrm{u}}}{M_{\mathrm{c}}} \tag{6}$$

deformability index $$D_J = \frac{\Phi_{\mathrm{u}}}{\Phi_{\mathrm{c}}} \tag{7}$$

overall index (J-factor) $$J = S_J \cdot D_J = \frac{M_{\mathrm{u}}}{M_{\mathrm{c}}} \cdot \frac{\Phi_{\mathrm{u}}}{\Phi_{\mathrm{c}}} \tag{8}$$

M_{u} is the ultimate moment, Φ_{u} is the ultimate curvature; M_{c} and Φ_{c} is the moment and curvature at the point of the maximum compressive strain of concrete beam is 0.001, viz. $\varepsilon_{\mathrm{c}} = 0.001$, before which the concrete can be regarded as linear elastic. Jaeger et al (1997) verified these indices by tests and found that there was a corresponded relation between J and the relative compressive depth of the beams. It can be seen for the moment-curvature curves that J is the ratio of the sums of the deformation energy and the complementary deformation energy where $\varepsilon_{\mathrm{c}} = 0.001$ acts as the characteristic point like the yield point in RC beams. J-factor is more reasonable because it covers the strength safety margin and the deformation safety margin of beams. It was adopted as a chapter, the design for deformability, in Canadian Highway Bridge Design Code (CHBDC). It is demanded that J should not be less than 4 for rectangular section beams and less than 6 for T-sections (Bakht 2000). Though J is useful for FRP reinforced concrete beams, it cannot be expanded to other materials even if RC beams because the characteristic concrete compressive strain is necessary but not current.

Deformability Index in Design Guide of ACI

In Guide for the Design and Construction of Concrete Reinforced with FRP Bars reported by ACI Committee 440 (2001), deformability index is listed in definitions and is defined as the ratio of energy absorption (area under the moment-curvature curve) at ultimate strength of the section to the energy absorption at service level, which is

$$D_{\mathrm{ACI}} = \frac{E_{\mathrm{u}}}{E_{\mathrm{s}}} \tag{9}$$

However, it is not used in the design program and the service level is not defined clearly. Less catastrophic failure with a higher deformation factor was observed when the member failed due to the crushing of concrete, which is qualitative. Though the deformability index is just a conception, it is considered by the strength reduction indices.

Ductility Index and Deformability Index in Design Recommendations of FHWA

In the research report of the Federal Highway Administration (FHWA) (Dolan et al 2001), ductility and deformability are discussed together for the concrete bridge beams prestressed by FRP tendons. Ductility is referred as the ratio of deflection at cracking (or initial yield for steel reinforcement) to ultimate deflection, which can be expressed as

$$\mu_{\Delta,\mathrm{FHWA}} = \frac{\Delta_{\mathrm{u}}}{\Delta_{\mathrm{cr}}} \quad \text{or} \quad \mu_{\Delta,\mathrm{FHWA}} = \frac{\Delta_{\mathrm{u}}}{\Delta_{\mathrm{y}}} \tag{10}$$

And the deformability index is defined as

$$D_{\mathrm{FHWA}} = \frac{\Phi_{\mathrm{u}}}{\Phi_{\mathrm{s}}} = \frac{(d-kd)\varepsilon_{FRP-Ultimate}}{(d-\frac{a}{\beta_1})\varepsilon_{FRP-Service}} \tag{11}$$

d, k,a and β_1 are the geometric parameters of the section, which are explained in the literature in detail. Φ_s is the curvature under the service level, which is defined as the load to produce a tensile stress in the concrete of $3\sqrt{f_c'}$.

This index is a function of the ratio of the ultimate strain to the prestressing strain with slight modification due to the differences in the neutral axis of elastic and inelastic behavior. It can be seen that the deformability index is nearly mimicked by the ratio of ultimate to initial prestress. In the recommendation, it is not used in design also.

Deformability Index Presented by Zou

A new definition of the deformability index for prestressed concrete beams, which is proposed by Zou (2003), is the ratio of the deflection at failure to the deflection at first cracking multiplied by the ratio of the ultimate moment (or load) to the cracking moment (or load)

$$Z = \frac{\Delta_{\mathrm{u}}}{\Delta_{\mathrm{cr}}} \cdot \frac{M_{\mathrm{u}}}{M_{\mathrm{cr}}} \tag{12}$$

There are 42 prestressed concrete beams calculated and compared using this index and the indices in Eqn.(1), Eqn.(4) and Eqn.(5). The Eqn.(4) is regarded as a reasonable indication of deformability for FRP prestressed beams while his index seems to be a suitable measurement of deformability for FRP and steel tendons. Actually, the physical conception of Z index is disordered because the deflection is a parameter for element but the moment is for a section. The index will be depended on the loading patterns and the supports condition of each element. However, it is referable that the cracking point is employed as the characteristic point for this index.

Deformability Index Presented by Tann et al

The deformability index of Tann et al (2004) is based on the RC beams strengthened by FRPP, which is

$$D_{\Delta,\mathrm{T}} = \frac{\Delta_{0.95}}{\Delta_{\mathrm{s}}} \tag{13}$$

$\Delta_{0.95}$, which is the deformation under 95% peak loads, is instead of Δ_u as the numerator in other definitions because the deformations under the ultimate load in test are uncertain due to the randomness. Δ_s is the service load which is suggested as the deformation under 67% of the design ultimate load. The ductility index defined as Eqn.(5) is also used in the paper. In addition, the failure modes of FRP-strengthened flexural members are classified into ductile, near ductile and brittle according to the deformability index and the ductility index. This index is suitable for design due to the simple calculation.

2.3 Robustness Indices

Basing on comparing the members' behavior of FRP and conventional materials, Van Erp (2001) presented that an element must has adequate robustness that should be considered in two sides: the over-loaded capacity and the deformable capacity. The robustness index that can describe the holistic performance was presented

$$R = S_{\mathrm{R}} \cdot D_R = \frac{S_{\mathrm{u}}}{S_{\mathrm{s}}} \cdot \frac{\Delta_{\mathrm{u}}}{\Delta_{\mathrm{s}}} \tag{14}$$

$S_{\mathrm{R}} = S_{\mathrm{u}} / S_{\mathrm{s}}$ is the ratio of the ultimate load carrying capacity over the serviceability limit state load, which is called strength index. $D_{\mathrm{R}} = \Delta_{\mathrm{u}} / \Delta_{\mathrm{s}}$ is the ratio of the deformation at failure over the deformation at the serviceability limit state, which is called deformability index. The robustness index *R* is similar to the index *J*, but the index *R* is universal.

2.4 Discussion about These Indices

The mentioned indices can be classified into four types according to the expression forms. The ratio of the generic deformations including deflections, curvatures and rotations, such as Eqn.(1), Eqn.(4), Eqn.(7), Eq(10), Eqn.(11) and Eqn.(13), is the commonest form because it can be acquired easily in tests. It is also a reasonable concept to use the deformation energy to define the overall performance, such as Eqn.(2), Eqn.(5) and Eqn.(9), because it is regarded as the essential variable. The strength is the most visible parameter to describe the safety margin of structure, so it is used in some indices, such as Eqn.(6) and the S_R index. The product of the deformations' ratio by the strengths' ratio, such as *J* index in Eqn.(8), *Z* index in Eqn.(12) and *R* index in Eqn.(14), reflects the overall performance of the elements, which have the concrete physical meaning and seem to be practicable in design according to the conclusions in the literatures. These four forms characterise the performance of the elements from different viewpoints. Therefore, they all should be considered in design for the various types of elements.

Most of these indices are the ratios of two states of the flexural elements. The difference between the two states is hinted as the safety margin of the elements which is what the performance index reflects. Definitions of these two states are the preconditions for defining the performance index. It is reasonable for all elements that the ultimate state acts as the state for the numerator if the randomness in test is neglected. However, it is difficult to define the other states because there are various types of elements even the same type of elements maybe have different failure modes initiated by the different materials as listed in Tab.1. To compare with the performance of all these elements and to reflect it all-sided, a uniform definition for the state in the denominator is needed.

Table 1 Flexure elements and their failure modes

Elments	Ab.	Materials	Failure modes	Failure materials
Steel beams	**SB**	Steel	The strain on the side reaches the ultimate strain.	Steel
All-FRP beams	**FB**	FRP	The strain on the side reached the ultimate strain.	FRP
Steel reinforced concrete beams	**RC**	Steel Concrete	1.The strain of the compressed concrete reaches the ultimate strain and the steel reinforcements have yielded.	Concrete, steel
			2.The strain of the compressed concrete reaches the ultimate strain and the steel reinforcements have not yielded.	Concrete
FRP reinforced concrete beams	**FC**	FRP Concrete	1.The strain of the compressed concrete reaches the ultimate strain and the FRP reinforcements have not broken.	Concrete
			2.FRP reinforcements break.	FRP
Prestressed concrete beams with FRP or high-strength steel tendons	**PC**	FRP/high-strength steel Concrete	1.The strain of the compressed concrete reaches the ultimate strain and the tendons have no failure.	Concrete
			2.The tendons failure.	FRP/high-strength steel

3. PROPOSAL OF PERFORMANCE INDECIES

We call the state in the denominator is the design-aimed state (DS), which is the upper limit of the service level. The distance between the actual ultimate loaded state (US) and the design-aimed state is the safety margin. Actually, the various design-aimed points in the previous literature have been defined. However, they are all based on the behavior of one type of elements, which leads to that they cannot be used to compare with each other. Hence, a recessive approach is proposed: present a set of performance indices firstly, and then find an appropriate index to determine the design-aimed states by the equal value of this index, finally check and adjust DS by tests data. Firstly, a set of performances indices are proposed as following

strength index $$S = \frac{M_u}{M_d} \tag{15}$$

deformability index $$D = \frac{\Phi_u}{\Phi_d} \tag{16}$$

deformation energy index(Y-factor) $$Y = \frac{E_u}{E_d} \tag{17}$$

overall index (F-factor) $$F = D^m \cdot S^n = \left(\frac{\Phi_u}{\Phi_d}\right)^m \cdot \left(\frac{M_u}{M_d}\right)^n \quad \textit{where} \quad m + n = 2 \tag{18}$$

where M_u, Φ_u and E_u are the moment (or generic load), the curvature (or generic deformation) and the deformation energy in the ultimate loaded state respectively, M_d, Φ_d and E_d are the parameters in the design-aimed state. *m* and *n* are called as the working condition coefficients whose values depend on the characteristic of the loads acting on the members. When the influence of the over loading is greater than the overstepping of deformation, $m<n$, whereas, $m>n$. In general, $m=n=1$, which is assumed in the following parts.

Modeling the load-deformation curve as two straight lines as the dash lines shown in Fig.7 approximatively, the following relations of the indices can be concluded

$$Y = D \cdot S - S + D \tag{19}$$

$$F = (Y + S - D)^m \cdot S^{(n-m)} = (Y + S - D)^n \cdot D^{(m-n)} \tag{20}$$

if $m=n=1$, $$F = Y + S - D \tag{21}$$

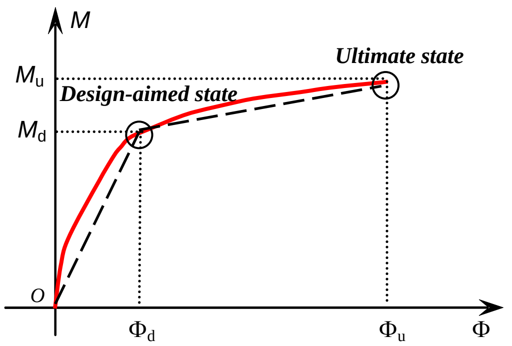


Figure 7 Two key states in load-deformation curve

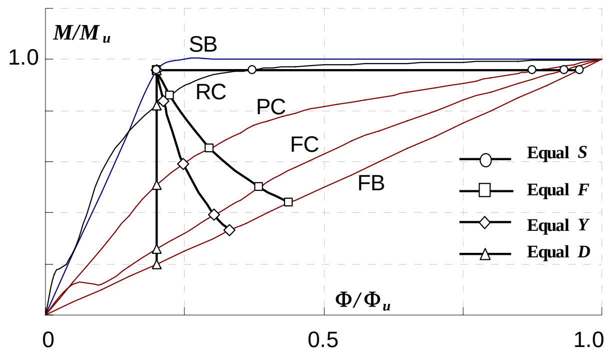


Figure 8 DS determined by proposed indices

Table 2 Indices in DS determined by proposed indices

Elements		SB	RC	PC	FC	FB
	S	**1.04**	1.22	1.96	3.88	5.00
D=5.00	*Y*	**9.17**	9.90	12.80	20.50	25.00
	F	**5.21**	6.10	9.80	19.40	25.00
	D	**5.00**	2.69	1.14	1.07	1.04
S=1.04	*Y*	**9.17**	4.46	1.29	1.15	1.09
	F	**5.21**	2.80	1.19	1.12	1.09
	D	**5.00**	4.72	4.03	3.30	3.03
Y=9.17	*S*	**1.04**	1.19	1.70	2.54	3.03
	F	**5.21**	5.63	6.83	8.41	9.17
	D	**5.00**	4.47	3.39	2.60	2.28
F=5.21	*S*	**1.04**	1.17	1.53	2.00	2.28
	Y	**9.17**	8.51	7.07	5.81	5.21

For varied types of elements listed in Tab.1, the load-deformation curves of typical element shown in Fig.8 can be achieved by normalizing them with the parameters in US. The curves are computed by the method of integrating section strips. The design-aimed point of SB is always fixed on the point when the most lager strain in the section reaches the yield strain. It has been an accepted conception. Based on it, the design-aimed points of the other elements can be determined when a certain performance index is equal to each element as shown in Fig.8, for example, the circle-shaped points are the calculated design-aimed points when *S* is 1.04 for each. Tab.2 lists the values of the indices on every condition.

It can be seen that there is not much difference for ordinary RC, which means the ductility index, that is *D*, has been able to reflect its overall performance. However, the differences are significant for the other members. Comparing all conditions, the same *S* will lead to over conservative value while a small safety margin will be led by the same *D*. When *Y* or *F* is same, it can be seen that *D* of PC, FC and FB are from 2.28 to 4.03 and the S from 1.53 to 3.03, all indices are valued in the reasonable range. Especially, when the *F* is equal, the design-aimed point of FB is near the middle point of the curve, which corresponds with that the safe index is 2.0 for the elastic material in the previous design methods. Therefore, *F* is suggested as the parameter to determine DS. In the future work, the DS should be checked and adjusted with tests data. In another viewpoint, it can be concluded that the safety storage for strength and the one for deformation can be exchanged each other without the overall safety margin loss according to the presented performance indices.

4. CONCLUSION

From the above analysis and results, the following conclusions can be drawn:
(a) The conventional ductility index should be updated into the deformability with the invention of new structure materials and the diversification of the performance of the flexural members.
(b) Many performance indices presented by previous researchers, which are reviewed, analyzed and compared in the paper and defined based on only one certain type of element. However, it is found that they are the ratios of two characteristic states of the elements. The design-aimed state, from which to the ultimate state is the safety margin, is presented and defined in this paper.
(c) A new approach to achieve the uniform performance indices and to determine the design-aimed state is proposed, which is also including the definition of the new performance indices, the calculation of the design-aimed state and the calibration with test data.
(d) A set of uniform performance indices, including the deformability index D, the strength index S, the deformation energy index Y and the overall performance index F are presented, which can reflect the performance of the flexural members in various viewpoints.
(e) By comparing the indices of five types of flexural members, F-factor is suggested to define the design-aimed point for all flexural members.
(f) The safety storage for strength and the one for deformation can be exchanged with each other without the overall safety margin loss according to the presented performance indices.

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